# The Johann Bernoulli Foundation: A Homage to Johann Bernoulli

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Ladies and Gentlemen,

On behalf of the Board of the Johann Bernoulli Foundation I welcome you to the 1991-1992 Johann Bernoulli Lecture. The Board hopes that it will be the first of a long sequence of lectures featuring prominent scientists, in particular, mathematicians. The objective of the Foundation is to promote mathematics in and around the University of Groningen, and one of the ways by which the Foundation seeks to attain this goal is to try and make these Johann Bernoulli Lectures into a yearly event.

I am glad that so many of you have accepted the open invitation to attend. No doubt this is because the lecturer this year is Professor Kalman, whom I bid a special welcome and thank for his willingness to deliver the first Johann Bernoulli lecture. After I have given a short presentation about Johann Bernoulli, I shall yield the floor to Professor Jan Willems, chairman of the Johann Bernoulli Foundation, who will introduce Professor Kalman.

Our Foundation is named after Johann Bernoulli because of his connection with the University of Groningen:
Johann, born precisely 325 years ago in 1667 in Basel, was appointed professor here in the "Mathematical Arts" [1] when he was 28, so in 1695, upon the recommendation of Christiaan Huygens, supported by the Marquis de l'Hôpital, the most outstanding French mathematician of those days. It was his first academic appointment. After 10 years, in 1705, Johann left Groningen and returned to his hometown Basel. On his way back he learned that his brother Jakob, who

*Slightly expanded version of the opening address on the occasion of the Johann Bernoulli Lecture 1991-1992 on June 3, 1992, organized by the Johann Bernoulli Foundation, founded in Groningen in 1988. The author is secretary of this Foundation.

[1] In his letter d.d. April 9, 1695 to Johann Bernoulli, Johannes Braun (then Rector of the University of Groningen) uses the term "Mathemathische Künsten"; see [8], p.406. The "Artes"-Faculty comprised, besides mathematics, a diversity of propaedeutic subjects including Greek, Hebrew, Philosophy, History, Biology, Logic, Astronomy, etc. There were three main Faculties in Groningen, namely those of Theology, Law and Medicine.

was 12 years older and professor in mathematics at the University of Basel, had died, and so he immediately succeeded Jakob in the chair.

If it had been broad day light, I would have asked you to take a closer look at the stained glass window on my righthand side, here in the corner. The window is called "Modern Times" and has been made by Johan Dijkstra in the years 1937–1951. But since there is not enough light outside to fully appreciate the window I show you instead this transparency of a detail of the window, copied from [3]. In this window you see a group of distinguished Groninger professors representing three centuries of scientific activities at the University of Groningen. The person on the lefthand side, the one holding a scroll, is Johann Bernoulli, and the little boy clutching at his gown is his son Daniel, born in Groningen in 1700; see [1] [2] and [3], pp. 20,21. Jakob (also called Jacques or James), Johann (Jean or John) and Daniel are the three most famous mathematicians of the Bernoulli family.

For an account of Johann's years in Groningen I refer to recent papers [6] and [7] of Gerard Sierksma of the Department of Econometrics of this University and the report [4] of the student seminar directed by Jan van Maanen of the Department of Mathematics, also of this University, which contains many further interesting details concerning the Bernoulli's and their work.

A second reason why the Foundation is named after Johann Bernoulli is that he may be considered as the very first calculus teacher, promoting what was then called the new method of calculating maxima and minima and tangents on the continent. In the following I want to argue this point. As sources I have used the books and papers mentioned in the references. I do not claim to make any original contribution: the remarks made here can be traced back to the cited literature. I found the manuscript [5] very inspiring.

In the second half of the 17-th century the infinitesimal calculus, in particular, the method to determine extremal values, had reached a glorious climax:
In 1665–1666, a few years before Johann was born, Isaac Newton had developed his calculus of fluxions. Independently, but somewhat later, when Johann was about 18, Gottfried Wilhelm Leibniz had published similar ideas in two papers in the Leipzig journal Acta Eruditorum, which from now on I shall call the Acta. In the Acta paper of 1684 he presented a new method to calculate maxima, minima and tangents of curves. It was the first paper on *differential* calculus. The other one of 1686 was the first paper on *integral* calculus.

Newton's method had its roots in kinematics involving time, velocity and acceleration. He did not publish the details of his work until 1704.

Leibniz' treatment was more geometrical and based on the characteristic tri-

[2] I am grateful to F.R.H. Smit, curator of the University Museum, for providing me with this reference.

angle of the infinitesimals $(dx, dy, ds)$, where $x$ and $y$ are the coordinates of a point on a curve and $s$ stands for arc length. Leibniz deliberately had kept his papers very obscure and cryptic, and, consequently, they encountered an almost universal lack of understanding; even Huygens had confessed that he could not follow the ideas of Leibniz; see [9], p. 131.

Johann learned mathematics from his older brother Jakob. Jakob studied theology at his father's wish, but as soon as possible he abandoned it in favor of mathematics which he had studied in his spare time. Also Johann made a false start in his career and studied medicine. But in his spare time Johann was introduced by Jakob to the mathematics of Descartes and Barrow, Newton's teacher, and mastered it "with an incredible speed", as Johann writes somewhere. Within a few years this teacher-student relation between the two brothers evolved in a relation of fellow students:
In 1687, when Johann was 20 and Jakob, then 32, had just been appointed professor of mathematics at the University of Basel, the brothers rediscover and work their way through the two Acta papers of Leibniz. Johann writes in his autobiography [10], p. 72, of 1741: "It was for us only a matter of a few days to unravel all its secrets." But we know that on December 15 of that year Jakob had written to Leibniz asking for clarification and explanation. These two accounts already indicate the difference between the dispositions of the two brothers; it will be amplified later on.

Leibniz was on one of his frequent political tours and received the letter only three years later. By then Jakob had solved the so called **Leibniz' Isochrone** (=equal time) problem, which Leibniz had posed in a paper in 1687:

*Find the shape of the curve along which a ball would roll down with uniform vertical velocity.*

Jakob solved the problem using the new method and published his results in the Acta in 1690. So Leibniz could be brief in his reply to Jakob's request for help: "You don't need any assistance from me, your paper shows that you have fully grasped the theory."

By 1690 Johann begins to have some influence on Jakob's work which shows that by that time they had become close collaborators:
(1) Johann provides Jakob with a proof that the harmonic series

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\dots$$

is divergent.
(2) Johann invents the name *integral* which Jakob uses in his Isochrone paper. It is the first proof of Johann's talent for finding appropriate terminology.
(3) In his Isochrone paper Jakob poses a problem suggested by Johann. It is the **Catenary** or Hanging Chain problem :

*"Find the shape assumed by a flexible chain suspended between two points and hanging under its own weight".*

It turned out later that this problem had been raised before by Galileo in his "Two New Sciences" in 1638. Galileo, incorrectly as Huygens had observed, concluded that the desired curve was a parabola.
The two brothers could not solve the problem. In the July 1690 issue of the Acta, Leibniz claims to have a solution, but postpones its publication till the end of the year to give other "geometers" a chance to produce a solution also. So the Bernoullis tried again and when finally the solution was obtained the rivalry between the two collaborators emerges. I let Johann continue the story: 28 years later, writing to Pierre Remond de Montmort (a French mathematician), he boasts (the translation is taken from [5]):
"The efforts of my brother were without success; for my part, I was more fortunate, for I found the skill (I say it without boasting, why should I conceal the truth?) to solve it in full and to reduce it to the rectification of the parabola. It is true that it cost me study that robbed me of rest for an entire night. It was much for those days and for the slight age and practice I then had, but the next morning, filled with joy, I ran to my brother, who was still struggling miserably with his Gordian knot without getting anywhere, always thinking like Galileo that the catenary was a parabola. Stop! Stop! I say to him, don't torture yourself any more to try to prove the identity of the catenary with the parabola, since it is entirely false. The parabola indeed serves in the construction of the catenary, but the two curves are so different that one is algebraic, the other is transcendental. I have unfolded the mystery. After having said all this, I showed him my solution and revealed the method which led me to it..."
Before the end of the year Johann informed the editor of the Acta that he had solved the problem and would send him the manuscript. It was published in June 1691 and was his first mathematical paper. It placed him in rank next to Leibniz and Huygens, who separately published correct solutions of the hanging chain problem in issues of the Acta of the same year. Of course, Huygens used the old method.

In 1690, at the age of 23, Johann begins to travel. Like his brother had done before, he goes to Geneva and France, where he teaches his recently mastered new mathematics. In Paris he meets the Marquis de l'Hôpital, who is very much impressed by the talents of the young visitor. As is well known the Marquis invites Johann to give him private lessons for a handsome fee at one his castles. Johann agrees and every other day during three months he delivers lectures and writes four pages of lecture notes which they discuss. In 1696 de l'Hôpital publishes his famous *Analyse des infiniments petits, pour l'intelligence des lignes courbes*, the first textbook on differential calculus. Included was a proof of what is now referred to as de l'Hôpital's rule concerning the limit $\frac{0}{0}$. Right after de l'Hôpital's death in 1704 Johann claimed that the book was almost identical with his French lecture notes and that it contained much from his letters to the

Marquis. Not until 70 years ago in 1922 could it be proved beyond doubt that Johann was right, when in Basel a copy of these lecture notes was found. Indeed, although the Marquis had made important corrections and improvements, the notes and the letters did provide him the basis for his book. So the famous rule of de l'Hôpital is actually due to Johann Bernoulli, and the book on differential calculus is essentially from Johann. In his later years in Basel he writes a sequel book on integral calculus. Thus Johann can be considered the author of the first comprehensive textbook on differential and integral calculus. It contains most of the material we offer nowadays to first year university students in their first analysis course. Even the notation is only slightly different from what we use today.

This makes Johann the first calculus teacher, but to be a good teacher he has to provide his students with many problems and examples to illustrate the power of the theory. In this Johann and Jakob worked very much together. In spite of themselves, they stimulated each other in posing and solving many problems. To say the least, their method was unorthodox. It was unprecedented and to my knowledge it has been copied only in much milder forms: They began to argue about priority rights and then they began to fight, first privately in their correspondence and then openly in their publications.

Their first fight begins when Johann is in France where he receives international recognition. For a year they correspond about, among many other problems, the **Velaria** problem:

*Establish the shape of a sail under the influence of the wind.*

Jakob sends Johann a differential equation of the second order without telling him where it comes from, and asks Johann to solve it, which Johann does in a few days. Johann recognises the differential equation as the one for the shape of the sail and thinks that Jakob cannot solve the problem. Johann publishes, without notifying his brother, a paper in the Journal des Sçavans in which he proves that the Velaria coincides with the curve of the hanging chain. He adds that the problem comes from his brother and that his brother could not solve it. Johann did not know that in the mean time Jakob had found a solution and had sent it to the Acta for publication. The rivalry, so far kept private in their correspondence, had come out in the open.

Johann is 25 when in 1692 he returns to Basel. Jakob is and remains irritated with Johann. Two years later, in a letter congratulating Johann with his doctor's thesis about muscle contraction, he reminds Johann that he, Johann, was his student and that everything Johann knows actually comes from him, Jakob.

The friction flares up again and becomes a public feud when Johann is in Groningen. From there in 1696 Johann challenges the mathematical community with his famous **Brachistochrone** (=shortest time) problem:

*Establish the curve between two fixed points along which a ball would roll in the shortest possible time.*

It aroused great interest and was very difficult. Leibniz solved it, and he wrote to Johann that besides those who were familiar with the new method, only Huygens, had he lived, and Johannes Hudde, Mayor of Amsterdam, had he continued with mathematics, could have solved the problem. He suggested to pose the problem again and to wait another half year for more solutions. So Johann did, and Leibniz was right. Only five solutions were submitted, namely by Johann, Jakob, Leibniz, de l'Hôpital and, of course, by Newton.

Johann's solution was the most elegant. He used Fermat's principle which states that a ray of light in different media travels along a path which requires the shortest time. Thus the ball would fall along the path a ray of light would take. The infinitesimal calculus comes in by considering infinitely many layers of different media of infinitesimal height.

Jakob's solution was rather clumsy and laborious, and in an open letter to Henri Basnage de Beauval (publisher of the "Histoire des Ouvrages des Sçavans", Rotterdam, 1687–1709) Johann made fun of it. What Johann failed to see was that Jakob's method was more general. It eventually led to the theory now known as the Calculus of Variations. Jakob realized the value of his approach and concluded his paper by posing a new variant of the well known **isoperimetric problem**:

*Determine the closed curve of a fixed length, which encloses the maximum area.*

This variant could only be attacked by using his version of the calculus of variations. Johann fell into the trap: He immediately published a solution but without derivation. It was only partially correct. Jakob could guess the method which his former student had employed and made a public bet:
(1) He could reproduce Johann's line of reasoning,
(2) he could pinpoint the mistakes in the derivation and
(3) he could give the right solution.
If he failed the first part he would pay 50 Imperials, double and triple the amount if he also failed in the second and third part of the bet. This started an acrimonious quarrel that dragged on for years. It got so bad that it became tragic for the brothers and embarrassing for the scientific communitiy. Pierre Varignon (professor in mathematics at the Collège Mazarin) wrote to both brothers and told them that they had been appointed members of the French Academy of Sciences, but that they had to stop their fight. It was to no avail, it had gone too far.

Perhaps stimulated by their ongoing rages of jealousy, Jakob and Johann had publicly posed many ingenious problems and solved them, at the same time challenging the mathematical community and inviting mathematicians to participate, not only in solving the problems but sometimes also in taking sides in the brotherly fights. In the more than 2800 letters and documents in the file of Johann more than 90 problems, including the Leibniz Isochrone, Catenary, Velaria and Brachistochrone, appear; see [8], pp. 515–518. Of course, not all of these were invented by the two Bernoullis. Often these problems were mathematically translated into differential equations which were then solved by ingeneous methods. In this way it was shown how powerful the new method was in comparison with the existing approaches.

Apparently Johann was the more cantankerous and the more impulsive of the two. He was also the more gifted, even brilliant at times. Jakob was more thoughtful, slower, often looking for a general result that could be extracted from studying special cases. Johann possessed the creative imagination and Jakob the critical depth. Together they formed the two sides of a scientific genius.

For the rest of his life Johann remained in Basel where he died at the age of 82. He initiated many students to the new method of Leibniz: There were, besides his sons Niklaus, Daniel and Johann, also Clairaut and Cramer and many others, but the greatest of them all was Leonhard Euler (whose father had been a theology student of Jakob).

After Newton's death in 1727 Johann was the nestor of the mathematicians. If Newton had had such a follower as Leibniz has had in Johann Bernoulli, the development of mathematics would most certainly have followed along different lines. It was not until the end of the eighteenth century that, through the efforts of Laplace, Newton's ideas were widely accepted on the continent. Johann truly was the first calculus teacher and, with the assistance of his brother, very effective at that.

Johann got many lucrative offers from other universities: from Utrecht, from Leiden and from Padua. In 1717 the University of Groningen invites him to come back and proposes a very tempting deal. Johann also declines this offer, but in his diaries [10], p 86, and [11], p 100, he concludes, that the Groningers must really have liked him and the way he fulfilled his professorship there. In those days many (but not all) did, and now we named a Foundation after him, a homage to a great teacher.

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