Laudatio for the Johann Bernoulli Lecture for Rudolf Kalman

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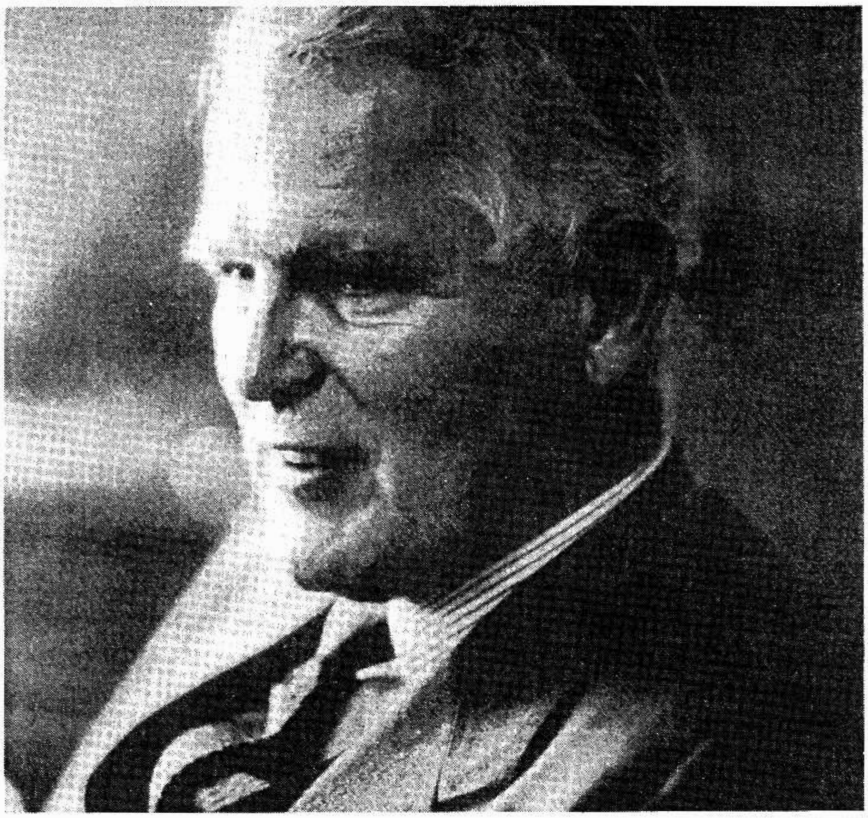
On behalf of the Johann Bernoulli Foundation, it is a great pleasure for me to introduce Professor Rudolf Kalman, our speaker for the 1991-92 Johann Bernoulli Lecture, and to tell you something about his work.

Let me start by reading you a few items from his Curriculum Vitae. Professor Kalman was born in Hungary in 1930. He moved to the United States as a youngster. He studied electrical engineering at the Massachusetts Institute of Technology as an undergraduate and obtained his Ph.D. from Columbia University. He subsequently worked at IBM and RIAS, a theoretical research department of the Martin-Marietta Aerospace Corporation. Following this he was professor at Stanford University. Presently he is Professor of Mathematical System Theory at the ETH in Zürich and Graduate Research Professor at the University of Florida. Professor Kalman holds several honorary degrees and obtained many international awards. Among these are the IEEE Medal of Honor, the Steele prize from the American Mathematical Society and, last but not least, the Kyoto Prize of the Inamori Foundation. This last prize is regarded as a Japanese version of the Nobel prize.

Professor Kalman is generally recognized as the founder of the field of Mathematical System Theory. Of course, there were earlier seeds to this field. Seeds in the form of single variable control theory, going back all the way to Maxwell, Routh, and Lyapunov, and to Bode, Black, and Nyquist. Seeds in the form of Pontryagin's maximum principle expressing the optimality of an open-loop controlled trajectory. Further, seeds in the form of a theory for the synthesis of electrical circuits heavily based on Heaviside's operational calculus. Finally, there were seeds in the form of the theory of filtering of signals from noise due to Wiener and Kolmogorov.

However, it was Kalman's seminal work which made systems and control theory to what it is today. By introducing a number of crucial and central concepts and by emphasizing the importance of mathematical thinking, he laid the foundations for a flourishing branch of applied mathematics. By always keeping in mind the issue of relevance – be it relevance to control engineering, to general science, or to econometric practice, he guaranteed that the field of system theory remained firmly imbedded in current scientific thinking.



R.E. KALMAN. Photograph from:
A.C. ANTOULAS, (ed.), *Mathematical Systems. Theory: The Influence of R.E. Kalman*, Springer Verlag 1991.

Kalman's early work is historically situated in electrical engineering. In the fifties and the sixties this field went through a true renaissance, a paradigm shift in the strict sense of the word. This paradigm shift can be characterized by a change in focus from what could be called *physics based thinking* to what could be called *information processing based thinking*. In physics based thinking the core curriculum of electrical engineering consisted of resistors, capacitors and inductors, of electrical machines and electromagnetism, of diodes, triodes and transistors. In information processing based thinking this core consists of information theory and signal processing, of systems and control theory, of computer programming and artificial intelligence. This paradigm shift did not occur without pain or resistance. Whereas in the United States this change of focus took place in the sixties, it took many Western European engineering departments until the eighties to make this change, often in a half-hearted way.

Two names were instrumental in achieving this paradigm shift. On the one hand, Claude Shannon with the development of information and coding theory. On the other hand, Rudolf Kalman with the development of systems and control theory. Those of us who were brought up as students in these branches of theoretical engineering experienced this change in a dramatic way. Before 1960 one could speak of control theory as a field with a clear practical relevance, scattered results, little cohesion, and with a diffuse scientific and mathematical basis. By 1970 this had completely changed. In 1970 we could speak of a discipline. A discipline with a number of central concepts: state, controllability, observability. A discipline with a number of strongly connected central problems: the linear-quadratic gaussian problem with the Kalman filter and the Riccati equation as its building blocks; the problem of adaptive stabilization; the problem of constructing state realization algorithms.

It was Kalman's work which shaped these problems. They remain central dominant themes in today's research. In order to appreciate this, it suffices to observe that the design of robust H-infinity controllers, the dominant theme in control theory research in the eighties, found its ultimate resolution in the discovery that it yields a feedback controller with a Kalman filter imbedded in it.

It would take me much too far to describe in detail the extent of Kalman's work. However, I would like to give you an overview of some of this main contributions and afterwards high-light a couple of these. Among Kalman's contributions we can recognize the following:

- *The Kalman filter*: A sophisticated algorithm for recursively estimating an unknown time-series from an observed one.
- The notions of *controllability* and *observability* – compelling concepts from any point of view. I will say a bit more about these later.

- The *Linear-Quadratic-Gaussian problem*: A universal procedure for designing a feedback controller.
- The theory of the *Riccati equation*, both the differential equation and the algebraic Riccati equation.
- The notion of the *state* of a dynamical system in an input/output context.
- *Realization theory*: algorithms for expressing an input/output dynamical system in state space form.
- The *algebraic* (module theoretic) *approach* to linear systems theory.
- The basic idea of *combining least squares parameter estimation with on line control* as a premise for adaptively controlling a system. This is still the main concept in adaptive control today.
- Algorithms for *estimating linear relations from noisy data*, and a critique of current econometric practice. I suspect that we will hear more about this later today during Kalman's own lecture.
- Kalman put forward already in 1956 ideas related to what nowadays is called *chaos*.

This is supposed to be but a brief introduction for a general audience and it is not the place to explain in a precise mathematical way what these various problems are or how they are resolved. Permit me however to elucidate briefly the notions of controllability and of observability, and the basic idea behind the Kalman filter.

The notion of controllability refers to the possibility of steering the state of a dynamical system from any initial to any terminal state by means of a suitable control action. If the control has sufficient influence to make this trajectory transfer possible, then the system is said to be *controllable*. One can see such trajectory selection questions come up in the most diverse applications, from patriots picking scuds out of the sky, to economists trying to steer economic variables to desirable values. Controllability is a necessary condition for the very existence of a suitable, of an optimal control. The notion of controllability was introduced by Kalman around 1960, and he derived conditions for controllability in terms of the parameters of a linear system. Nowadays these results are practically the first ones which are taught in any control course anywhere in the world.

The notion of observability can be explained as follows. Suppose that we have a dynamical system, a black box, and suppose that it produces two signals, two time trajectories. The first one is observed, the second one is not. If it is possible to deduce the second trajectory, the latent one, from the first, the manifest one, and from the laws governing the system, then we call the system *observable*.

One can again see this type of question come up in the most diverse areas, from a chemical engineer trying to deduce a difficult to measure concentration from temperature and pressure measurements, to phychologists trying to deduce a latent feature of the psyche from a manifest one.

The dynamical algorithm by means of which the unobserved signal is deduced from the observed one is called an *observer*. The notion of observability, as I just now formulated it, is basically an existential question. It tells us when it is ideally possible to deduce the unobserved signal from the observed one. However in practice these ideal circumstances will not be met, due, for example, to the following complications:

1. The black box may be driven by unknown noisy inputs. This noise will make it impossible to predict the unobserved signal exactly.
2. The observations may also be noisy. Thus the measured signal will be imbedded in spurious noise and we should be cautious not to put too much faith in these measurements.
3. In trying to deduce the unobserved signal from the observed one by means of an on-line algorithm we need an automatic data reduction procedure. Irrelevant past measurements should be automatically discarded. Otherwise our data processing device will irrevocably run out of memory.

The first two considerations bring us to the problem of the design of optimal observers in the presence of plant driving noise and of measurement noise. It is customary to talk about a *filter* in this case. Indeed, the problem is to filter the irrelevant, the noisy information out of the observations, so that as much as possible only the useful information remains. Filtering problems attracted a great deal of attention during and just after the second World War. They were studied by the US mathematician Norbert Wiener and by his Soviet colleague Kolmogorov, two of the most prominent mathematicians of the the 20-th century. Independently they developed a filtering algorithm, referred to as the Wiener-Kolmogorov filter. The Wiener-Kolmogorov algorithm attracted great attention both among mathematicians and engineers. Kalman's work which be referred to as entitled 'New methods in Wiener Filtering Theory' totally eclipsed this attention. The Kalman filter has a number of advantages which make it far superior indeed. Crucial is that it starts from a convenient mathematical model of the measured and the to - be - reconstructed signal. This model is what in system theory we call a state space model. The resulting Kalman filter algorithm has a number of important features, to wit:

- *It is recursive* in the observations. This yields the desired data reduction algorithm. The filter automatically stores the relevant information from the past measurements and allows to discard the past measurements.
- It is eminently suitable for computer implementation.

- The filter has a very appealing *cybernetic structure*, consisting of an internal model for the plant with a feedback loop around it. Through this feedback, the observed innovations and the filter gain take care of updating the internal model. Permit me to skip the details.
- It offers a convenient algorithm for computing the parameter matrices of the filter in terms of the parameter matrices of the plant. This algorithm is based on the so-called *Riccati equation*, a quadratic matrix equation which has provided the type of mathematical puzzle which is a must for any respectable branch of normal science.

It is perhaps ironic that the contempories Johann Bernoulli and Jacopo Riccati, two pioneers of calculus, should meet today in a lecture about control theory. Or perhaps it is not. After all, Johann Bernoulli is often credited with the principle of optimality, the key idea in dynamic programming and optimal feedback control, and Count Riccati has his name associated with the bottle-neck equation of modern control theory.

Back to the Kalman filter. The Kalman filtering algorithm had a truly unbelievable impact. Statements to this effect abound. From high level officials in the US research establishment calling the Kalman filter the best return ever on research money invested, to NASA engineers doubting whether the moon program could have been carried out without it.

Dear Professor Kalman, you must, by now, feel like a man who fell into deep water and who just before drowning sees his whole life flashing by. Appropriate to the occasion, my account of your work has emphasized the historical aspects. Unavoidably, my account of your work has been a subjective one. I know that it is hard to hear other people tell about one's own work. However, you will now be given ample opportunity for rectifications and additions. I would hereby like to yield this cathedra to you and ask you kindly to deliver the 1991-92 Johann Bernoulli lecture.

Selected publication list of R.E. Kalman

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