The Work of Professor Sir Christopher Zeeman FRS

Floris Takens *
Department of Mathematics, University of Groningen
P.O. Box 800, 9700 AV Groningen, The Netherlands

On behalf of the Johann Bernoulli Foundation of Mathematics, it is a great pleasure for me to introduce Professor Sir Christopher Zeeman, our speaker for the 1992-1993 Johann Bernoulli Lecture, and tell you something about his work.

CURRICULUM

Christopher Zeeman was born in 1925, served as a flying officer in the Royal Air Force from 1943 to 1947, was a scholar at Cambridge, Christ's College from 1947 to 1953 and received his PhD in 1954. From 1955 to 1964 he was lecturer at Cambridge University and from 1964 to 1988 he was foundation professor and director of the research centre at the University of Warwick. In this last period he created the Mathematics Department of the newly founded University of Warwick and turned it, in spite of a financial climate which was often adverse to universities, into one of the top institutes in England. Since 1988 he is Gresham Professor of Geometry and principle of Hertford College at Oxford.

During his career he received many honours among which eight honorary doctor degrees, and was invited to many prestigeous lectures before the present Johann Bernoulli Lecture.

SCIENTIFIC WORK

In his scientific work, there are two main themes: the first is algebaic and geometric topology, on which he published during the period from 1954 to 1968; the second is catastrophe theory, dynamical systems and chaos with their applications to physical, biological and behavioural sciences, on which he published from 1968 on. Around 1968, and also occasionally later he published various works on brain modelling. Though these themes are quite different, one thing which all his works have in common is a strong appeal to geometric intuition, and to intuition in general.

ALGEBRAIC AND GEOMETRIC TOPOLOGY

Although the first theme, algebraic and geometric topology, is hard to describe

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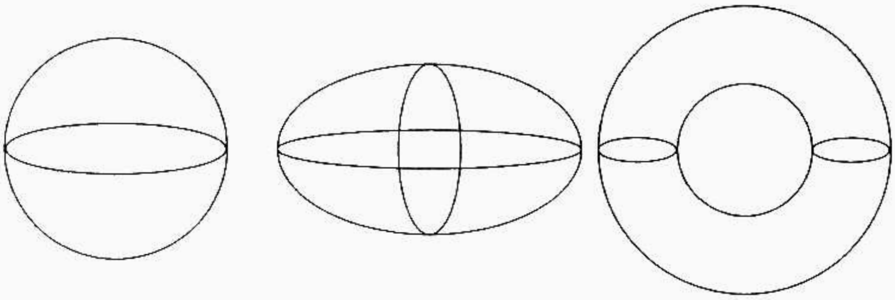
[*] Introduction of the main speaker for the Johann Bernoulli Lecture 1992–193 on February 16, 1993, organized by the Johann Bernoulli Foundation for Mathematics, founded in Groningen in 1988.

to a non-mathematical audience I hope to be able to make clear at least what the general context and relevance of the problems considered are.

In many mathematical problems one is given an equation, and one is supposed to give the 'relevant' information about its solutions. A first question is then: how many solutions are there? If the answer is zero, then the problem is completely solved; if the answer is one, the next question may be to determine the solution, or a reasonable approximation of it; often however there are many solutions. A simple example is $x^2 + y^2 + z^2 = 1$, where we are interested in solutions (x, y, z) consisting of three real numbers. Clearly there are many solutions, so many that it does not make sense to try to make a list of them. Still the set of all these solutions has a simple structure: it is represented by the unit sphere in 3-space. Other examples are:

- $x^2 + 2y^2 + 3z^2 = 1$: an ellipsoid;
- $(x^2 + z^2 - 4)^2 + y^2 = 1$: a torus.

These three different cases are illustrated in Figure 1 below.



sphere ellipsoid torus

FIGURE 1.

In all three cases we have surfaces in the 3-dimensional space. They are different, but in some sense the latter is more different: it is possible to deform a sphere continuously into any ellipsoid (continuous deformation: deformation without cutting and pasting, but distances may get distorted), but it is impossible to continuously deform a sphere into a torus. This is a subtle point, since each small part of a torus can be continuously deformed to a small part of a sphere and vice versa. In this situation we say that globally the torus and the sphere are different, even up to deformations.

One may ask what is the relevance of these global considerations. To explain this we need to observe that whenever we analyse the possible solutions of an equation, we need to know what elements are admitted as solutions. It turns out that sometimes we have to consider equations, where the solutions have to lie on some surface like the ones we saw before (we give a simple example below) and it happens that the number of solutions of such equations is influenced by the global properties of the surface.

EXAMPLE. We may think of the surface of the earth as a sphere, and of the wind as defining a continuous velocity field on the sphere (and tangent to the sphere). We consider the problem of finding a point *on the sphere* where the wind velocity is zero (one can put this in the form of an equation, the solutions of which must be points in the surface of the sphere). According to a mathematical theorem, this problem always has at least one solution (independent of the weather). This result however is no longer true if the sphere is replaced by a torus. See Figure 2.

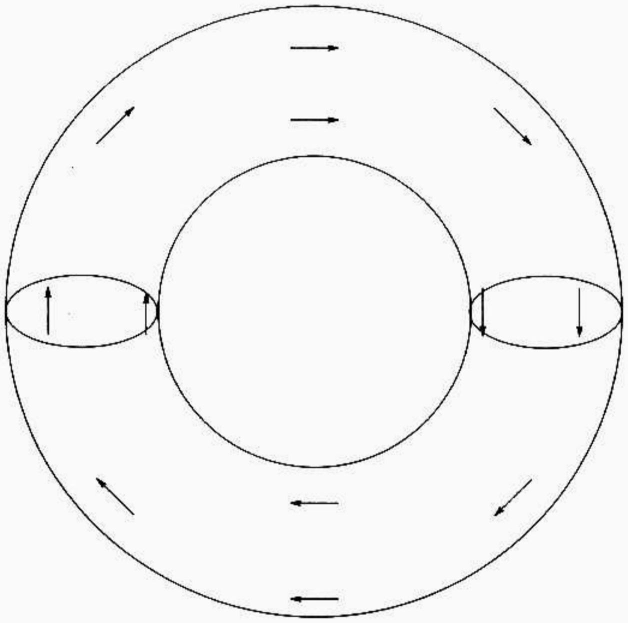


FIGURE 2. A continuous velocity field on a torus

So these mysterious 'global properties' have quite concrete consequences when it comes to solving equations which are defined on such surfaces. The investigation of the effect of such global properties on the existence of solutions for many types of equations is now an important research area in the overlap between analysis and topology.

It is to the understanding of these global aspects of surfaces, or rather of their higher dimensional analogues, that the research efforts of Professor Zeeman were directed in this first period. In fact, around 1960 there was a strong and concentrated effort, by a number of the greatest mathematicians of this century, to get a better understanding of these global poperties and one of the great challenges was to solve the generalized Poincaré conjecture. Henri Poincaré was a famous french mathematician who lived around 1900, who also tried to understand these global problems, but who did not get very far, due to the state of development of mathematics at that time. Although this Poincaré conjecture is still not solved completely, all the major break throughs were marked by solving the conjecture for certain special cases, one of these being due to our present guest: In 1961 he solved the so called combinatorial Poincaré conjecture in the dimensions larger than 4. Since these questions cannot be explained much further without considering spaces of dimensions larger than 3, I now turn to the second theme of research.

CATASTROPHE THEORY AND RELATED SUBJECTS

In order to describe the basic ideas of catastrophe theory, we come back to analysing the solutions of equations as one of the central activities in mathematics. One can describe catastrophe theory as the theory which deals with the question how solutions change when an equation changes, and in particular how continuous changes in an equation can provoke discontinuous changes in the solutions. In some cases, like buckling of elastic structures, such discontinuities were already investigated, but the contribution of catastrophe theory was the recognition of the fact that there are certain underlying mathematical principles which organize these discontinuities in a way which is often quite independent of the context in which they occur. These ideas originated from the French mathematician René Thom who used them to describe, in a rather philosophical way, many phenomena in biology, mainly in embryology. Sir Christopher Zeeman realized that the possibilities of applications could be carried over to other domains of science as well. On the one hand he started to give interpretations in terms of catastrophe theory of a great variety of different phenomena, ranging from the stability of ships to prison disturbances and from the stock market instabilities to behavioural instabilities under stress. On the other hand he extended the original, very much simplifying, assumptions of catastrophy theory so as to make the theory applicable to a greater variety of cases. Before going on I like to give an example of the type of discontinuities which one considers.

EXAMPLE. I refer to a phenomenon which can be observed when enormous clouds of starlings are flying during the preparation for migration. In these clouds one sees dark regions some of whose edges give the impression of sharp discontinuities — an example is shown in Figure 3. A possible explanation is the following. The starlings tend to fly in such a way that they are all in or near a smooth and slightly curved suface. The density which we see in any direction depends on the number of times the line through our eye in that direction intersects the surface of the birds and on the angles of these intersections. Where this direction is tangent to the surface of the birds, one sees the sharp discontinuities. It is then a mathematical theorem that in the 'generic' (or non-

FIGURE 3. A cloud of starlings (photo by drs. H. de Boer)

exceptional) situation the only patterns which the discontinuities can form are the fold, the cusp, and an intersection of two fold lines which can also be seen (in a much sharper way) in a numerically calculated picture of a projection of the torus, as presented in Figure 4.

In this example it was not hard to translate the situation to a mathematical framework. The lists of possible discontinuities is then obtained by the mathematical machinery of catastrophe theory. This translation into mathematics is often not so straightforward, for example in the behavioural sciences.

The general principle, underlying many of the applications made by Zeeman of the principles of catastrophe theory, is that it is not necessary to know the precise details of the mathematical model if one only wants information on the types of discontinuities. In this way it is quite legitimate to extrapolate from the discontinuities in the stability of ships, say due to displacements of cargo, to dicontinuities in human or animal behaviour, say due to changes of parameters like stress and fear.

In other examples Zeeman extended the original catastrophe theory, or, as it is now often called, elementary catastrophe theory, so as to be able to deal with situations which are more complex: although the precise details of the model are not so relevant for the type of singularities, the *structure* of the model is very important. By considering these more complex situations, he was able to connect the work in catastrophe theory with the theory of singular perturbations (in his work on heartbeat and nerve impulse), the theory of dynamical systems and chaos (in his work on turbulence), the classical theory

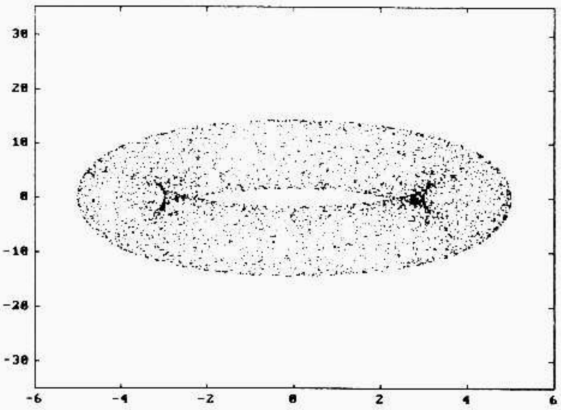


FIGURE 4. Foldlines and cusps in a projection of a torus

of differential equations (in his work on Duffing's equation in brain modelling), and with diffusion equations (in his recent work on the stability of dynamical systems) — I do not pretend to be complete here nor elswhere in this lecture.

The influence of these applications were considerable: they contributed to a more open attitude towards applications of mathematics. Exaggerating somewhat, one can say that where applied mathematics used to be confined to investigate the equations, and their solutions, given by the accepted mathematical models for the different phenomena, the work of Zeeman showed a much more liberal attitude towards the choice of these models. In this sense he paved the way for abstract parts of mathematics, to the applications. This was in particular true for singularity theory, which is the mathematical basis of catastrophe theory, and for the theory of dynamical systems, which is the mathematical basis of chaos theory.

PERSONAL REMARKS

Dear Christopher, I finally want to mention a few aspects of your work which impressed me when visiting your institute in Warwick and which were not mentioned before. Most important is the inspiring atmosphere. In the first place this was due to the enthousiastic and stimulating way in which you took part in many discussions, not only mathematical ones. In part this was also due to the many visiting mathematicians which you were always able to invite, due to a magic ability of attracting financial resources, and also to the fact that you were able to get the building of your institute constructed in such a way that interaction between staff members is strongly promoted: a large common room, with blackboards, centrally situated, close to the library. I think that all these aspect were very important to the succes of the institute in Warwick.

Finally, I think it must be hard to listen to someone else talking about your own work. However you will be given now ample opportunity for rectifications. I yield this cathedra to you and ask you kindly to deliver the 1992-1993 Johann Bernoulli Lecture.